

## Geometry

### Standard – Processes or Content Strand

#### CLE – Course Level Expectation

#### *SPI – State Performance Indicator*

✓ – Check for Understanding

### Standard 1 – Mathematical Processes

**CLE 3108.1.1 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely in mathematical reasoning.**

*SPI 3108.1.1 Give precise mathematical descriptions or definitions of geometric shapes in the plane and space.*

✓3108.1.4 Recognize that a definition depends on undefined terms and on previous definitions.

**CLE 3108.1.2 Apply and adapt a variety of appropriate strategies to problem solving, including testing cases, estimation, and then checking induced errors and the reasonableness of the solution.**

*SPI 3108.1.4 Use definitions, basic postulates, and theorems about points, lines, angles, and planes to write/complete proofs and/or to solve problems.*

*SPI 3108.1.2 Determine areas of planar figures by decomposing them into simpler figures without a grid.*

✓3108.1.1 Check solutions after making reasonable estimates in appropriate units of quantities encountered in contextual situations.

✓3108.1.9 Expand analysis of units of measure to include area and volume.

**CLE 3108.1.3 Develop inductive and deductive reasoning to independently make and evaluate mathematical arguments and construct appropriate proofs; include various types of reasoning, logic, and intuition.**

*SPI 3108.1.4 Use definitions, basic postulates, and theorems about points, lines, angles, and planes to write/complete proofs and/or to solve problems.*

✓3108.1.6 Use inductive reasoning to write conjectures and/or conditional statements.

✓3108.1.13 Use proofs to further develop and deepen the understanding of the study of geometry (e.g. two-column, paragraph, flow, indirect, coordinate).

✓3108.1.14 Identify and explain the necessity of postulates, theorems, and corollaries in a mathematical system.

**CLE 3108.1.4 Move flexibly between multiple representations (contextual, physical written, verbal, iconic/pictorial, graphical, tabular, and symbolic), to solve problems, to model mathematical ideas, and to communicate solution strategies.**

*SPI 3108.1.3 Use geometric understanding and spatial visualization of geometric solids to solve problems and/or create drawings.*

✓3108.1.2 Determine position using spatial sense with two and three-dimensional coordinate systems.

✓3108.1.10 Use visualization, spatial reasoning, and geometric modeling to solve problems.

✓3108.1.11 Identify and sketch solids formed by revolving two-dimensional figures around lines.

**CLE 3108.1.5 Recognize and use mathematical ideas and processes that arise in different settings, with an emphasis on formulating a problem in mathematical terms, interpreting the solutions, mathematical ideas, and communication of solution strategies.**

✓3108.1.3 Comprehend the concept of length on the number line.

**CLE 3108.1.6 Employ reading and writing to recognize the major themes of mathematical processes, the historical development of mathematics, and the connections between mathematics and the real world.**

✓3108.1.8 Understand how the similarity of right triangles allows the trigonometric functions sine, cosine, and tangent to be defined as ratio of sides.

✓3108.1.12 Connect the study of geometry to the historical development of geometry.

**CLE 3108.1.7 Use technologies appropriately to develop understanding of abstract mathematical ideas, to facilitate problem solving, and to produce accurate and reliable models.**

✓3108.1.5 Use technology, hands-on activities, and manipulatives to develop the language and the concepts of geometry, including specialized vocabulary (e.g. graphing calculators, interactive geometry software such as Geometer's Sketchpad and Cabri, algebra tiles, pattern blocks, tessellation tiles, MIRAs, mirrors, spinners, geoboards, conic section models, volume demonstration kits, Polydrons, measurement tools, compasses, PentaBlocks, pentominoes, cubes, tangrams).

✓3108.1.7 Recognize the capabilities and the limitations of calculators and computers in solving problems.

**Standard 2 – Number & Operations**

**CLE3108.2.1 Establish the relationships between the real numbers and geometry; explore the importance of irrational numbers to geometry.**

*SPI 3108.2.1 Analyze, apply, or interpret the relationships between basic number concepts and geometry (e.g. rounding and pattern identification in measurement, the relationship of pi to other rational and irrational numbers)*

✓3108.2.1 Analyze properties and aspects of pi (e.g. classical methods of approximating pi, irrational numbers, Buffon's needle, use of dynamic geometry software).

✓3108.2.2 Approximate pi from a table of values for the circumference and diameter of circles using various methods (e.g. line of best fit).

✓3108.2.6 Analyze precision, accuracy, and approximate error in measurement situations.

**CLE3108.2.2 Explore vectors as a numeric system, focusing on graphic representations and the properties of the operation.**

*SPI 3108.2.2 Perform operations on vectors in various representations.*

✓3108.2.3 Recognize and apply real number properties to vector operations and geometric proofs (e.g. reflexive, symmetric, transitive, addition, subtraction, multiplication, division, distributive, and substitution properties).

✓3108.2.4 Add vectors graphically and algebraically.

✓3108.2.5 Multiply a vector by a scalar graphically and algebraically.

**CLE3108.2.3 Establish an ability to estimate, select appropriate units, evaluate accuracy of calculations and approximate error in measurement in geometric settings.**

**Standard 3 – Algebra**

**CLE 3108.3.1 Use analytic geometry tools to explore geometric problems involving parallel and perpendicular lines, circles, and special points of polygons.**

*SPI 3108.3.1 Use algebra and coordinate geometry to analyze and solve problems about geometric figures (including circles).*

*SPI 3108.3.2 Use coordinate geometry to prove characteristics of polygonal figures.*

✓3108.3.1 Prove two lines are parallel, perpendicular, or oblique using coordinate geometry.

✓3108.3.2 Connect coordinate geometry to geometric figures in the plane (e.g. midpoints, distance formula, slope, and polygons).

✓3108.3.3 Find the equation of a circle given its center and radius and vice versa.

✓3108.3.4 Apply the midpoint and distance formulas to points and segments to find midpoints, distances, and missing information in two and three dimensions.

**CLE 3108.3.2 Explore the effect of transformations on geometric figures and shapes in the coordinate plane.**

*SPI 3108.3.3 Describe algebraically the effect of a single transformation (reflections in the x- or y-axis, rotations, translations, and dilations) on two-dimensional geometric shapes in the coordinate plane.*

- ✓3108.3.5 Use mapping notation to identify the image of a transformation given the coordinates of the pre-image.
- ✓3108.3.6 Identify a transformation given its mapping notation.

## Standard 4 – Geometry & Measurement

### **CLE 3108.4.1 Develop the structures of geometry, such as lines, angles, planes, and planar figures, and explore their properties and relationships.**

#### ***SPI 3108.4.1 Differentiate between Euclidean and non-Euclidean geometries.***

- ✓3108.4.1 Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true and discuss unique properties of each.

#### ***SPI 3108.4.2 Define, identify, describe, and/or model plane figures using appropriate mathematical symbols (including collinear and non-collinear points, lines, segments, rays, angles, triangles, quadrilaterals, and other polygons).***

- ✓3108.4.3 Solve problems involving betweenness of points and distance between points (including segment addition).
- ✓3108.4.4 Describe and recognize minimal conditions necessary to define geometric objects.
- ✓3108.4.5 Use vertical, adjacent, complementary, and supplementary angle pairs to solve problems and write proofs.
- ✓3108.4.6 Describe the intersection of lines (in the plane and in space), a line and a plane, or of two planes
- ✓3108.4.7 Identify perpendicular planes, parallel planes, a line parallel to a plane, skew lines, and a line perpendicular to a plane.
- ✓3108.4.8 Apply properties and theorems about angles associated with parallel and perpendicular lines to solve problems.
- ✓3108.4.21 Use properties of and theorems about parallel lines, perpendicular lines, and angles to prove basic theorems in Euclidean geometry (e.g., two lines parallel to a third line are parallel to each other, the perpendicular bisectors of line segments are the set of all points equidistant from the endpoints, and two lines are parallel when the alternate interior angles they make with a transversal are congruent).
- ✓3108.4.22 Perform basic geometric constructions using a straight edge and a compass, paper folding, graphing calculator programs, and computer software packages (i.e., bisect and trisect segments, congruent angles, congruent segments, a line parallel to a given line through a point not on the line, angle bisector, and perpendicular bisector).

### **CLE 3108.4.2 Describe the properties of regular polygons, including comparative classification of them and special points and segments.**

#### ***SPI 3108.4.3 Identify, describe and/or apply the relationships and theorems involving different types of triangles, quadrilaterals and other polygons.***

- ✓3108.4.9 Classify triangles, quadrilaterals, and polygons (regular, non-regular, convex and concave) using their properties.
- ✓3108.4.10 Identify and apply properties and relationships of special figures (e.g., isosceles and equilateral triangles, family of quadrilaterals, polygons, and solids).
- ✓3108.4.11 Use the triangle inequality theorems (e.g., Exterior Angle Inequality Theorem, Hinge Theorem, SSS Inequality Theorem, Triangle Inequality Theorem) to solve problems.
- ✓3108.4.12 Apply the Angle Sum Theorem for polygons to find interior and exterior angle measures given the number of sides, to find the number of sides given angle measures, and to solve contextual problems.
- ✓3108.4.14 Identify and use medians, midsegments, altitudes, angle bisectors, and perpendicular bisectors of triangles to solve problems (e.g., find segment lengths, angle measures, points of concurrency).

**CLE 3108.4.3 Develop an understanding of the tools of logic and proof, including aspects of formal logic as well as construction of proofs.**

***SPI 3108.4.4 Analyze different types and formats of proofs.***

- ✓3108.4.2 Compare and contrast inductive reasoning and deductive reasoning for making predictions and valid conclusions based on contextual situations.
- ✓3108.4.15 Identify, write, and interpret conditional and bi-conditional statements along with the converse, inverse, and contra-positive of a conditional statement.
- ✓3108.4.16 Analyze and create truth tables to evaluate conjunctions, disjunctions, conditionals, inverses, contra-positives, and bi-conditionals.
- ✓3108.4.17 Use the Law of Detachment, Law of Syllogism, conditional statements, and bi-conditional statements to draw conclusions.
- ✓3108.4.18 Use counterexamples, when appropriate, to disprove a statement.
- ✓3108.4.19 Use coordinate geometry to prove properties of plane figures.
- ✓3108.4.20 Prove key basic theorems in geometry (i.e., Pythagorean Theorem, the sum of the angles of a triangle is 180 degrees, characteristics of quadrilaterals, and the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length).

**CLE 3108.4.4 Develop geometric intuition and visualization through performing geometric constructions with straightedge/compass and with technology.**

**CLE 3108.4.5 Extend the study of planar figures to three-dimensions, including the classical solid figures, and develop analysis through cross-sections.**

***SPI 3108.4.5 Describe solids and/or surfaces in three-dimensional space when given two-dimensional representations for the surfaces of three-dimensional objects.***

- ✓3108.4.13 Locate, describe, and draw a locus in a plane or space (e.g., fixed distance from a point on a plane, fixed distance from a point in space, fixed distance from a line, equidistant from two points, equidistant from two parallel lines, and equidistant from two intersecting lines).
- ✓3108.4.23 Describe the polyhedron or solid that can be made from a given net including the Platonic Solids.
- ✓3108.4.24 Develop and use special formulas relating to polyhedra (e.g., Euler's Formula).
- ✓3108.4.25 Use properties of prisms, pyramids, cylinders, cones, spheres, and hemispheres to solve problems.

**CLE 3108.4.6 Generate formulas for perimeter, area, and volume, including their use, dimensional analysis, and applications.**

***SPI 3108.4.6 Use various area of triangle formulas to solve contextual problems (e.g., Heron's formula, the area formula for an equilateral triangle and  $A = \frac{1}{2} ab \sin C$ ).***

***SPI 3108.4.7 Compute the area and/or perimeter of triangles, quadrilaterals and other polygons when one or more additional steps are required (e.g. find missing dimensions given area or perimeter of the figure, using trigonometry).***

***SPI 3108.4.8 Solve problems involving area, circumference, area of a sector, and/or arclength of a circle.***

- ✓3108.4.28 Derive and use the formulas for the area and perimeter of a regular polygon. ( $A = \frac{1}{2} ap$ )

**CLE 3108.4.7 Apply the major concepts of transformation geometry to analyzing geometric objects and symmetry.**

***SPI 3108.4.10 Identify, describe, and/or apply transformations on two and three dimensional geometric shapes***

- ✓3108.4.31 Use properties of single transformations and compositions of transformations to determine their effect on geometric figures (e.g. reflections across lines of symmetry, rotations, translations, glide reflections, and dilations).
- ✓3108.4.32 Recognize, identify and apply types of symmetries (point, line, rotational) of two- and three- dimensional figures.

✓3108.4.33 Use transformations to create and analyze tessellations and investigate the use of tessellations in architecture, mosaics, and artwork.

✓3108.4.34 Create and analyze geometric designs using rigid motions (compositions of reflections, translations, and rotations).

**CLE 3108.4.8 Establish processes for determining congruence and similarity of figures, especially as related to scale factor, contextual applications, and transformations.**

*SPI 3108.4.11 Use basic theorems about similar and congruent triangles to solve problems.*

✓3108.4.35 Prove that two triangles are congruent by applying the SSS, SAS, ASA, AAS, and HL congruence statements.

✓3108.4.36 Use several methods, including AA, SSS, and SAS, to prove that two triangles are similar.

✓3108.4.37 Identify similar figures and use ratios and proportions to solve mathematical and real-world problems (e.g., Golden Ratio).

✓3108.4.38 Use the principle that corresponding parts of congruent triangles are congruent to solve problems.

*SPI 3108.4.12 Solve problems involving congruence, similarity, proportional reasoning and/or scale factor of two similar figures or solids.*

✓3108.4.29 Extend the effect of a scale factor  $k$  in similar objects to include the impact on volume calculations and transformations.

**CLE 3108.4.9 Develop the role of circles in geometry, including angle measurement, properties as a geometric figure, and aspects relating to the coordinate plane.**

*SPI 3108.4.13 Identify, analyze and/or use basic properties and theorems of circles to solve problems (including those relating right triangles and circles).*

✓3108.4.39 Identify lines and line segments associated with circles.

✓3108.4.40 Find angle measures, intercepted arc measures, and segment lengths formed by radii, chords, secants, and tangents intersecting inside and outside circles.

✓3108.4.41 Use inscribed and circumscribed polygons to solve problems concerning segment length and angle measures.

**CLE 3108.4.10 Develop the tools of right triangle trigonometry in the contextual applications, including the Pythagorean Theorem, Law of Sines and Law of Cosines**

*SPI 3108.4.9 Use right triangle trigonometry and cross-sections to solve problems involving surface areas and/or volumes of solids.*

✓3108.4.26 Describe and draw cross-sections (including the conic sections) of prisms, cylinders, pyramids, spheres, and cones.

✓3108.4.27 Use right triangle trigonometry to find the area and perimeter of quadrilaterals (e.g. square, rectangle, rhombus, parallelogram, trapezoid, and kite).

✓3108.4.30 Use right triangle trigonometry to find the lateral area (if possible), surface area, and volume of prisms, cylinders, cones, pyramids, spheres, and hemispheres.

*SPI 3108.4.14 Use properties of right triangles to solve problems (such as involving the relationship formed when the altitude to the hypotenuse of a right triangle is drawn).*

✓3108.4.42 Use geometric mean to solve problems involving relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.

✓3108.4.43 Apply the Pythagorean Theorem and its converse to triangles to solve mathematical and contextual problems in two- or three-dimensional situations.

✓3108.4.44 Identify and use Pythagorean triples in right triangles to find lengths of an unknown side in two- or three-dimensional situations.

✓3108.4.45 Use the converse of the Pythagorean Theorem to classify a triangle by its angles (right, acute, or obtuse).

✓3108.4.46 Apply properties of  $30^\circ - 60^\circ - 90^\circ$  and  $45^\circ - 45^\circ - 90^\circ$  to determine side lengths of triangles.

***SPI 3108.4.15 Determine and use the appropriate trigonometric ratio for a right triangle to solve a contextual problem.***

- ✓3108.4.47 Find the sine, cosine and tangent ratios of an acute angle of a right triangle given the side lengths.
- ✓3108.4.48 Define, illustrate, and apply angles of elevation and angles of depression in real-world situations.
- ✓3108.4.49 Use the Law of Sines (excluding the ambiguous case) and the Law of Cosines to find missing side lengths and/or angle measures in non-right triangles.

## **Standard 5 – Data Analysis, Statistics, & Probability**

**CLE 3108.5.1 Analyze, interpret, employ and construct accurate statistical graphs.**

***SPI 3108.5.1 Use area to solve problems involving geometric probability (e.g. dartboard problem, shaded sector of a circle, shaded region of a geometric figure).***

- ✓3108.5.3 Estimate or calculate simple geometric probabilities (e.g., number line, area model, using length, circles).
- ✓3108.5.1 Determine the area of each sector and the degree measure of each intercepted arc in a pie chart.
- ✓3108.5.2 Translate from one representation of data to another (e.g., bar graph to pie graph, pie graph to bar graph, table to pie graph, pie graph to chart) accurately using the area of a sector.

**CLE 3108.5.2 Develop the basic principles of geometric probability.**