Content in Focus
Geometry

In order to achieve the content and rigor of the new standards, teachers will need to learn how to compress many topics into a few lessons – called *rich tasks* – and extract many objectives from those.

Also, the notion that topics must be introduced in the timeline given by a textbook is generally not ideal. Students are capable of, prefer to, and should combine topics that a typical textbook would consider separately.

The typical geometry teacher follows the textbook. Solids are at the end of the book. But is this the most advantageous way in which to teach the objectives?

What if, early in the school year, students are asked to construct geometric solids, like cylinders, cones, prisms, pyramids, etc.? Students could study two and three-dimensional objects simultaneously. And instead of introducing the formulas for volume first, have students fill their constructed 3-dimensional objects with popcorn or sand or birdseed and measure the volume using graduated cylinders from science class. The Pythagorean theorem, trigonometry, measuring angles, and many more objectives could be covered in just a few constructions. Also, do not think that students need to do that same task everyday until it is mastered. Introduce it, then move to another topic, then re-visit the original topic – only this time, take it deeper. Here are a few of the CLEs and Checks for Understanding that could be covered by such an approach.

CLE	3108.4.1	Develop the structures of geometry, such as lines, angles, planes, and planar figures, and explore their properties and relationships.
CLE	3108.4.2	Describe the properties of regular polygons, including comparative classification of them and special points and segments.
CLE	3108.4.4	Develop geometric intuition and visualization through performing geometric constructions with straightedge/compass and with technology.
CLE	3108.4.5	Extend the study of planar figures to three-dimensions, including the classical solid figures, and develop analysis through cross-sections.
CLE	3108.4.6	Generate formulas for perimeter, area, and volume, including their use, dimensional analysis, and applications.

✓	3108.1.11	Identify and sketch solids formed by revolving two-dimensional figures around lines.
✓	3108.1.1	Check solutions after making reasonable estimates in appropriate units of quantities encountered in contextual situations.
✓	3108.1.2	Determine position using spatial sense with two and three-dimensional coordinate systems.

- ✓ 3108.1.5 Use technology, hands-on activities, and manipulatives to develop the language and the concepts of geometry, including specialized vocabulary (e.g. graphing calculators, interactive geometry software such as Geometer's Sketchpad and Cabri, algebra tiles, pattern blocks, tessellation tiles, MIRAs, mirrors, spinners, geoboards, conic section models, volume demonstration kits, Polydrons, measurement tools, compasses, PentaBlocks, pentominoes, cubes, tangrams).
- ✓ 3108.1.8 Understand how the similarity of right triangles allows the trigonometric functions sine, cosine, and tangent to be defined as ratio of sides.
- ✓ 3108.1.9 Expand analysis of units of measure to include area and volume.
- ✓ 3108.1.10 Use visualization, spatial reasoning, and geometric modeling to solve problems.

In this respect, we can often begin at the end of the textbook and work backward. Most athletes don't learn the rules by reading the rulebook; they learn the rules by playing the game with a referee. Solids can be taught *simultaneously* with two-dimensional geometry. And constructions on a Cartesian plane (graph paper) lend themselves well to analytic geometry.

Another *rich task* that is often overlooked or placed in the category of interesting, but not necessary – especially in a curriculum packed with more material than last year – is that of *tessellations*. Tesselations can provide a forum to discuss almost all of the properties of lines, segments, and polygons – even three-dimensional objects can be studied through tessellations.

The use of construction and study is vital for the geometry teacher who wishes for his students to master the curriculum and enjoy doing so. Just give students straightedge and compass, and give them a project to construct. For example: construct a circle with radius 3.2 cm, and a 30 degree inscribed angle, and a central angle that contains or intersects that same arc.

And for teachers who do not feel creative, there are many books and ancillary materials on this subject, as well as a wealth of expert teachers who have many ideas they would love to share – all they have to do is ask.

Content in Focus
Algebra II

In order to achieve the content and rigor of the new standards, teachers will need to learn how to compress many topics into a few lessons – called *rich tasks* – and extract many objectives from those.

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The typical geometry teacher follows the textbook. Solids are at the end of the book. But is this the most advantageous way in which to teach the objectives?

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✓	3108.1.11	Identify and sketch solids formed by revolving two-dimensional figures around lines.
✓	3108.1.1	Check solutions after making reasonable estimates in appropriate units of quantities encountered in contextual situations.
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✓	3108.1.5	Use technology, hands-on activities, and manipulatives to develop the language and the concepts of geometry, including specialized vocabulary (e.g. graphing calculators, interactive geometry software such as Geometer's Sketchpad and Cabri, algebra tiles, pattern blocks, tessellation tiles, MIRAs, mirrors, spinners, geoboards, conic section models, volume demonstration kits, Polydrons, measurement tools, compasses, PentaBlocks, pentominoes, cubes, tangrams).

✓ 3108.1.8 Understand how the similarity of right triangles allows the trigonometric functions sine, cosine, and tangent to be defined as ratio of sides.
✓ 3108.1.9 Expand analysis of units of measure to include area and volume.
✓ 3108.1.10 Use visualization, spatial reasoning, and geometric modeling to solve problems.

The Algebra II curriculum is full of concepts that are more of an esoteric nature, and as a result, concrete examples are harder to come by for some of these than in Geometry or Algebra I. As in no other math course, Algebra II creates more challenges for students and teachers alike to utilize applications of transcendental and imaginary numbers with which students can relate.

Although *real life* examples for imaginary and complex numbers certainly exist, they are not in the domain of the common experience for most people. Imaginary/complex numbers crop up in electrical engineering, stock market (the Black-Scholes option pricing model), number theory, fluid mechanics, and other places, but they are not everyday occurrences for the rank-and-file. For this reason, students should be encouraged to find models when they exist and are within the scope of this course, but also to study the topics without always needing to find applications – at least immediately.

Much of the Algebra II course should involve students discovering how various functions and their inverses operate. Students should be encouraged to utilize graphing utilities, such as a graphing calculator or computer graphing application, so that many graphs and their transformations can be graphed in a short period of time to allow the student time to see what effect a particular multiplication or addition or exponential or logarithmic operation has on a function. Reflecting a function through the line $y = x$ allows the student to look at a geometric transformation that generates the graph of an inverse.

✓ 3103.3.3 Determine and graph the inverse of a function with and without technology.
✓ 3103.3.4 Analyze the effect of changing various parameters on functions and their graphs.
SPI 3103.3.7 Identify whether a function has an inverse, whether two functions are inverses of each other, and/or explain why their graphs are reflections over the line y = x.

Instead of lecturing on trig functions, have students construct the unit circle, graph radii at 15 degree increments ($\frac{\pi}{12}$ radians), measure the rectangular coordinates on graph paper by hand, record the data as cos A and sin A, respectively, record the values of $\frac{y}{x}$ as tan A, etc., then allow students to graph the angle versus the sin A, cos A, and tan A on separate graphs, along with their reciprocals, csc A, sec A, and cot A.

CLE 3103.4.1 Understand the trigonometric functions and their relationship to the unit circle.
CLE 3103.4.2 Know and use the basic identities of sine, cosine, and tangent as well as their reciprocals.
✓ 3103.1.7 Use the unit circle to determine the exact value of trigonometric functions for commonly used angles (0°, 30°, 45°, 60°...).

This discovery approach works for e^x, *ln(x)*, and other functions that comprise the body of the Algebra II course. Discrete mathematics, with its arithmetic and geometric sequences, are best learned in a lab situation, and with student-led discovery of the various formulas instead of teachers and textbooks presenting them in a lecture or printed format before students have a chance to discover them.

✓ 3103.1.8 Understand and describe the inverse relationship between exponential and logarithmic functions.

✓ 3103.1.9 Translate the syntax of technology to appropriate mathematical notation for non-linear and transcendental functions.

Conic sections can be as concrete as you like. For example, pour flavored gelatin in a pointed ice cream cone, pop it out on a plate, and cut with a fine wire. Students can make their own conic sections before studying them formally. Large parabolic reflectors can be made out of cardboard and aluminum foil and used as solar reflectors to boil water, or as a listening device.

SPI 3103.3.11 Graph conic sections (circles, parabolas, ellipses and hyperbolas) and understand the relationship between the standard form and the key characteristics of the graph.

In other words, allow the students time to explore the concepts *before* they see them in a textbook or the instructor lectures on the topics. This *guided discovery* approach is the best and fastest way for students to gain a feel for the nuances of the functions presented in the Algebra II curriculum.