Developing Mathematical Thinking Through Questioning

What is an effective question?

•It is a question that probes for deeper meaning and often sets the stage for further questioning.

•It fosters the development of critical thinking skills and higher order capabilities.

•It promotes problem-solving and understanding.

•A good question is the principle component of designing inquiry-based learning.

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If you only ask for answers, you might get the correct response, but no assurance that the student understands the concept.

For example: What is 2^2 ?

The student says "4".

What if the student is thinking:
"Exponent multiplies base"
Answer is 4.

Of course, this student will also say that $3^2 = 6$!

Can you really answer questions correctly and have no clue what is happening?

Developing Mathematical Thinking Through Questioning

Almost everyone scores 100% on this test. However, no one knows anything about Traxoline, nor do they care.

Even so, if this had been a real quiz, each person would have received an "A."

That is why we must be careful when using multiple-choice questions or questions that are simply rote memorization with no indication of understanding. This is also why it is important to ask students to show their work and why you should give partial credit when the correct process is there.

Studying for a test or quiz can be a game in many ways, and many students have learned to "play" without actually absorbing any knowledge.

Most students can answer these questions. Students and teachers alike have sets of well-developed strategies for producing correct answers to questions that they do not understand.

While this is sometimes a useful skill, we aspire to more in our mathematics classes.

(Seizing Opportunities, AAAS, 1997)

Developing Mathematical Thinking with Effective Questions

To help students build confidence and rely on their own understanding, ask…

- Why is that true?
- How did you reach that conclusion?
- Does that make sense?
- Can you make a model to show that?

To help students learn to reason mathematically, ask…

- Is that true for all cases? Explain.
- Can you think of a counterexample?
- How would you prove that?
- What assumptions are you making?

To check student progress, ask…

- Can you explain what you have done so far? What else is there to do?
- Whey did you decide to use this method?
- Can you think of another method that might have worked?
- Is there a more efficient strategy?
- What do you notice when…?
- Why did you decide to organize your results like that?
- Do you think this would work with other numbers?
- Have you thought of all the possibilities? How can you be sure?

To help students collectively make sense of mathematics, ask…

- What do you think about what _____ said?
- Do you agree? Why or why not?
- Does anyone have the same answer but a different way to explain it?
- Do you understand what _____ is saying?
- Can you convince the rest of us that your answer makes sense?

To encourage conjecturing, ask…

- What would happen if…? What if not?
- Do you see a pattern? Can you explain the pattern?
- What are some possibilities here?
- Can you predict the next one? What about the last one?
- What decision do you think he/she should make?

To promote problem solving, ask…

- What do you need to find out?
- What information do you have?
- What strategies are you going to use?
- Will you do it mentally? With pencil and paper? Using a number line?
- Will a calculator help?
- What tools will you need?
- What do you think the answer or result will be?

Developing Mathematical Thinking with Effective Questions, cont.

To help when students get stuck, ask…

- How would you describe the problem in your own words?
- What do you know that is not stated in the problem?
- What facts do you have?
- How did you tackle similar problems?
- Could you try it with simpler numbers? Fewer numbers? Using a number line?
- What about putting things in order?
- Would it help to create a diagram? Make a table? Draw a picture?
- Can you guess and check?
- Have you compared your work with anyone else? What did other members of your group try?

To make connections among ideas and applications, ask…

- How does this relate to…?
- What ideas that we have learned before were useful in solving this problem?
- What uses of mathematics did you find in the newspaper last night?
- Can you give me an example of…?

To encourage reflection, ask…

- How did you get your answer?
- Does you answer seem reasonable? Why or why not?
- Can you describe your method to us all? Can you explaining why it works?
- What if you had started with… rather than…?
- What if you could only use…?
- What have you learned or found out today?
- Did you use or learn any new words today? What did they mean? How do you spell them?
- What are the key points or big ideas in this lesson?

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Table 3-2

Aligning Achievement Targets to Assessment Methods

Target to Be Assessed	*Assessment Method*			
	Selected Response	Extended Written Response	Performance Assessment	Personal Communication
Knowledge Mastery	Good match for assessment mastery of elements of knowledge.	Good match for tapping understanding of relationships among elements of knowledge.	Not a good match—too time consuming to cover everything.	Can ask questions, evaluate answers and infer mastery—but a time-consuming option.
Reasoning Proficiency	Good match only for assessing understanding of some patterns of reasoning.	Written descriptions of complex problem solutions can provide a window into reasoning proficiency.	Can watch students solve some problems and infer reasoning proficiency.	Can ask student to "think aloud" or can ask followup questions to probe reasoning.
Skills	Not a good match. Can assess mastery of the knowledge prerequisites to skillful performance, but cannot rely on these to tap the skill itself.		Good match. Can observe and evaluate skills as they are being performed.	Strong match when skill is oral communication proficiency; not a good match otherwise.
Ability to Create Products	Not a good match. Can assess mastery of the knowledge prerequisites to the ability to create products, but cannot assess the quality of products themselves.	Strong match when the product is written. Not a good match when the product is not written.	Good match. Can assess the attributes of the product itself.	Not a good match.

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Descriptive or Evaluative Feedback

Mark each example of **descriptive feedback** with a ***D*** and each example of **evaluative feedback** with an ***E***. If you believe it is **neither**, mark it with an ***X***.

_____ Good job!

_____ Sloppy work

_____ How did you reach that conclusion? Where's your data?

_____ Proficient

_____ ☺

_____ Your calculations are accurate. Take another look at appropriate units for density.

_____ C-

_____ Excellent!

_____ You need to try harder next time. You can do it!

_____ The students at station two are ready for the lab, they have their books cleared and their safety glasses on.

_____ ✯

_____ You need to label the x-axis, include units with your label, choose an appropriate scale, show the points you plotted, and give the graph a title.

_____ 81%

Self Assessment Task

I did these really well:

1.

2.

I could have:

1.

2.

Next time I need to focus on:

1.

2.

Peer Assessment Task

You did these really well:

1.

2.

You could have:

1.

2.

Next time you need to focus on:

1.

2.